Advanced Algebra 2 - 2x2 determinants

If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - c \cdot b$, find …

1. $\begin{vmatrix} 5 & 3 \\ 2 & -4 \end{vmatrix}$

$= 5(-4) - (3)(2)$
$= -20 - 6$
$= \boxed{-26}$

2. $\begin{vmatrix} 2 & 3 \\ -20 & -4 \end{vmatrix}$

$= 2(-4) - (3)(-20)$
$= -8 + 60$
$= \boxed{52}$

3. $\begin{vmatrix} 5 & 2 \\ 2 & -20 \end{vmatrix}$

$= 5(-20) - (2)(2)$
$= -100 - 4$
$= \boxed{-104}$

4. $\begin{vmatrix} -4 & 2 \\ -5 & 3 \end{vmatrix}$

$= (-4)(3) - (2)(-5)$
$= -12 + 10$
$= \boxed{-2}$

Solving 2x2 Systems Using Cramer's Rule

You can use determinants to solve a system of linear equations. The method, called **Cramer's rule** and named after the Swiss mathematician Gabriel Cramer (1704–1752), uses the **coefficient matrix** of the linear system.

LINEAR SYSTEM	COEFFICIENT MATRIX
$ax + by = e$ $cx + dy = f$	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

CRAMER'S RULE FOR A 2 X 2 SYSTEM

Let A be the coefficient matrix of this linear system:

$$ax + by = e$$
$$cx + dy = f$$

If $\det A \neq 0$, then the system has exactly one solution. The solution is:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A} \quad \text{and} \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

Solving 2x2 Systems

Example 1 - Watch- do not take notes yet...

$7x - 5y = 11$

$3x + 10y = -56$

$D = \begin{vmatrix} 7 & -5 \\ 3 & 10 \end{vmatrix}$ $= 7(10) - (-5)(3)$ $= 70 - (-15)$ $= 85$

$D_x = \begin{vmatrix} 11 & -5 \\ -56 & 10 \end{vmatrix}$ $= 11(10) - (-5)(-56)$ $= 110 - 280$ $= -170$

$D_y = \begin{vmatrix} 7 & 11 \\ 3 & -56 \end{vmatrix}$ $= 7(-56) - (11)(3)$ $= -392 - 33$ $= -425$

Answer:

$x = \frac{D_x}{D} = \frac{-170}{85} = -2$

$y = \frac{D_y}{D} = \frac{-425}{85} = -5$

$(-2, -5)$

check

$7(-2) - 5(-5) \stackrel{?}{=} 11$

$-14 + 25 = 11$

$11 = 11$ ✓

check

$3(-2) + 10(-5) \stackrel{?}{=} -56$

$-6 - 50 = -56$

$-56 = -56$ ✓

Solving 2x2 Systems

Example 2- What needs to be done first???? Put in STANDARD FORM!

$y = -2x + 3$ → $2x + y = 3$ (replace for D_x)

$5x + 6y = 4$

Answer: (replace for D_y)

$$D = \begin{vmatrix} 5 & 6 \\ 2 & 1 \end{vmatrix} = 5(1) - 6(2) = 5 - 12 = -7$$

$$D_x = \begin{vmatrix} 4 & 6 \\ 3 & 1 \end{vmatrix} = 4(1) - 6(3) = 4 - 18 = -14$$

$$D_y = \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = 5(3) - (4)(2) = 15 - 8 = 7$$

$$x = \frac{D_x}{D} = \frac{-14}{-7} = 2 \qquad y = \frac{D_y}{D} = \frac{7}{-7} = -1 \qquad (2, -1)$$

Check (ORIGINAL)

$y = -2x + 3$

$-1 \stackrel{?}{=} -2(2) + 3$

$-1 = -4 + 3$

$-1 = -1$ ✓

Check

$5x + 6y = 4$

$5(2) + 6(-1) \stackrel{?}{=} 4$

$10 - 6 = 4$

$4 = 4$ ✓